Masked Circuit Compiler in the Cardinal Random Probing Composability Framework

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CryptoExperts

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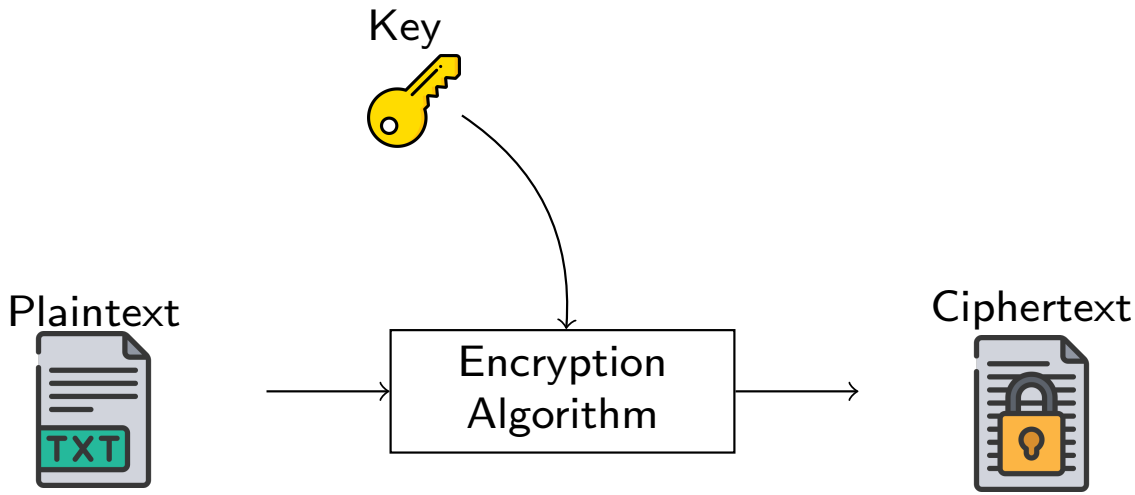
Side Channel Attacks



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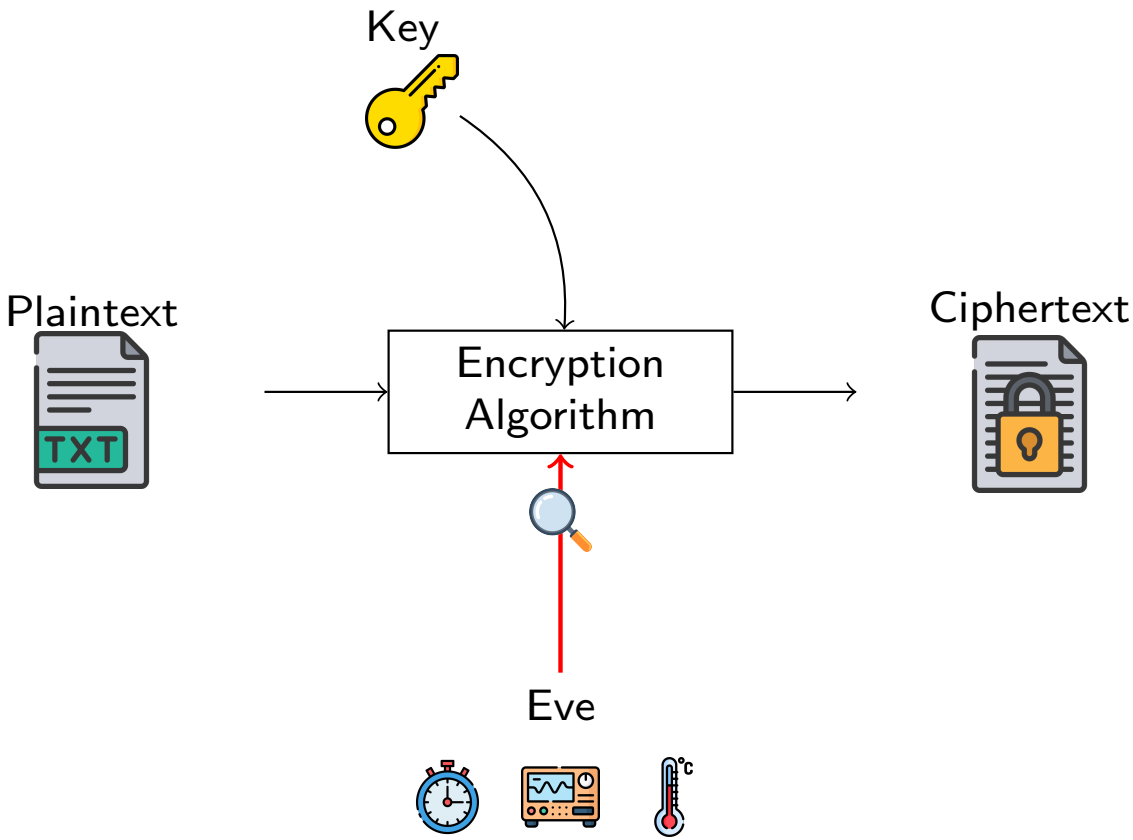

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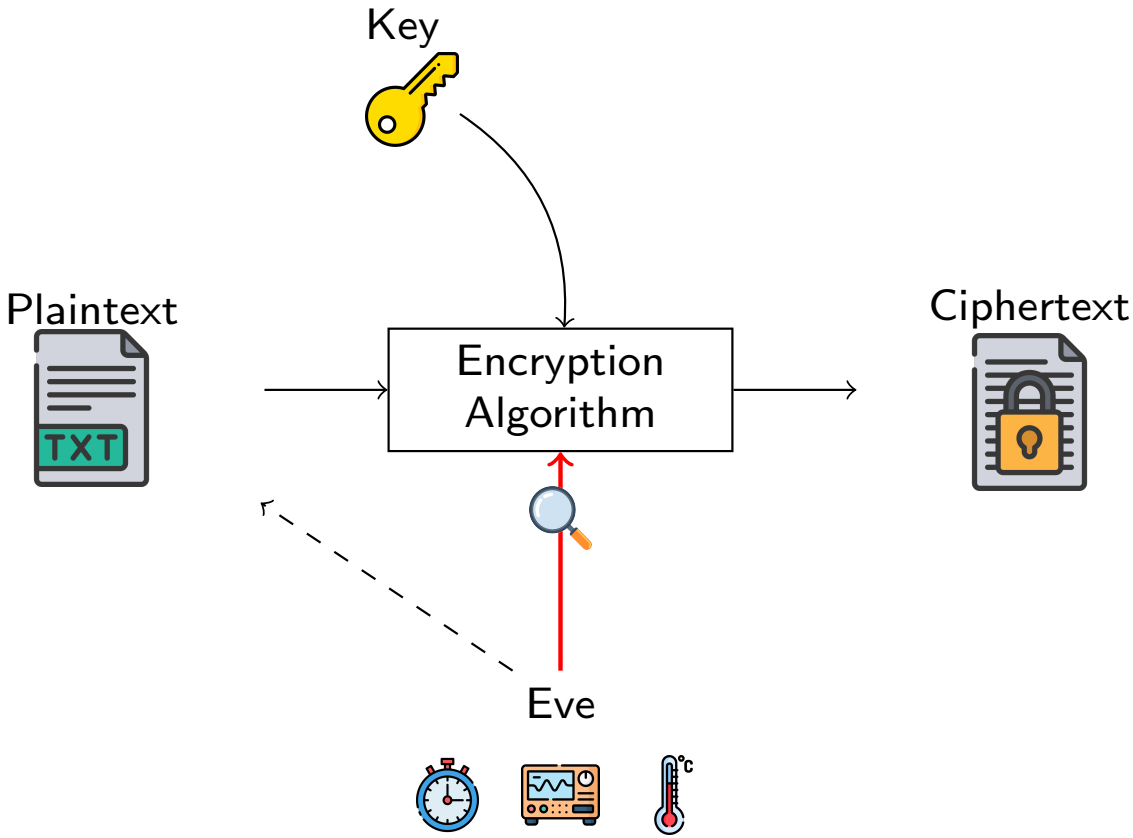


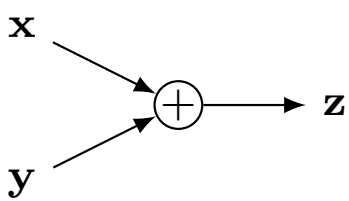
Masking countermeasure

- Replace a sensitive value $\mathbf{x}$ of the Algorithm by a tuple $(\mathbf{x}_1, \ldots, \mathbf{x}_n)$.
- Any set of $(n-1)$ shares is independent from $\mathbf{x}$.
- Arithmetic masking :

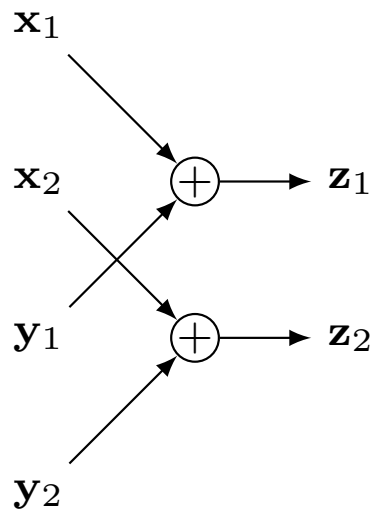
$$\mathbf{x} = \sum_{i=1}^{n} \mathbf{x}_i$$

Gadget

- Operations on sensitives values $\Longrightarrow$ Operations on their respective shares.

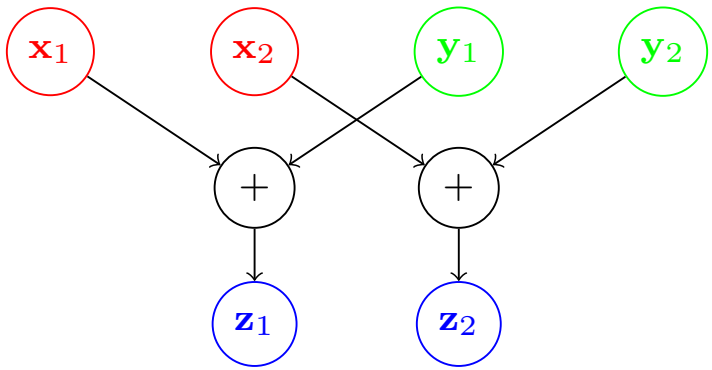


(a) Non masked version



(b) Masked version with 2 shares.

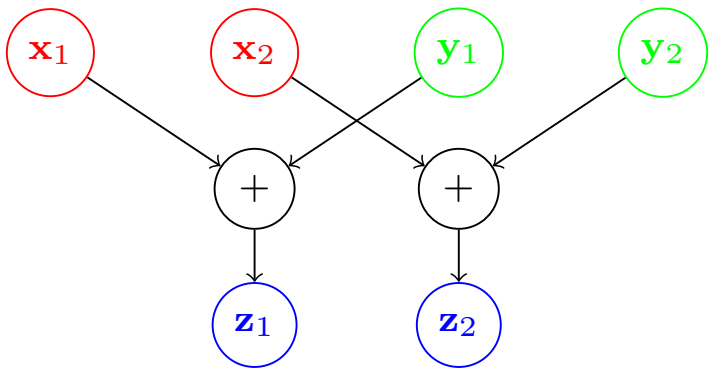
Circuits



Circuits

- Directed acyclic graph.
- Vertex : Arithmetic gate *or* input/output share.
- Edge : Wire of the circuit.

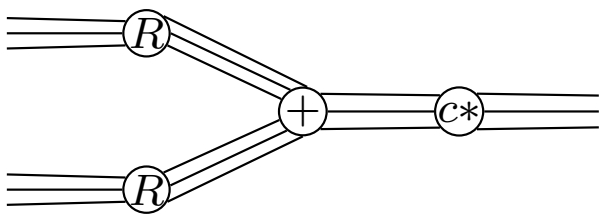
(p,ε)-Random probing model



(p, ε)-Random probing model

- Every wires leak its value with a probability $p \in [0, 1]$.
- Secure if there is a negligible probability ($\leq \varepsilon$) to get information on the secrets.

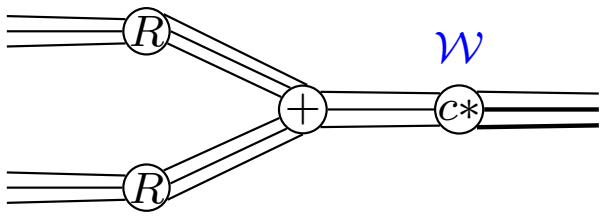
Threshold Random Probing Composability



(t, p, ε)-RPC :

- **Simulate** the internal leakage $\mathcal{W}$ and t output shares with **at most** t input shares of each secret variable.
- Error probability ε.

Threshold Random Probing Composability



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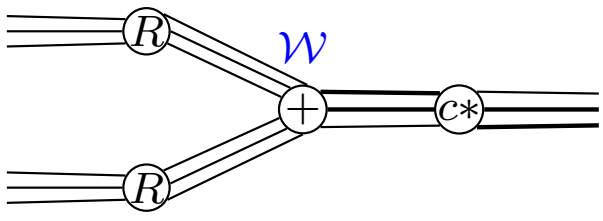

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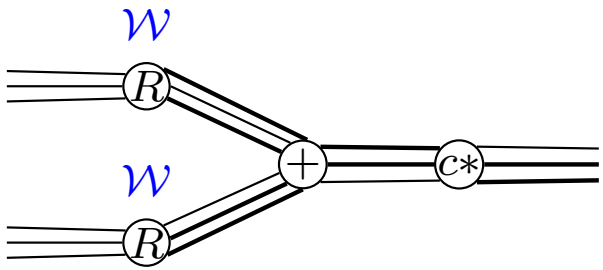

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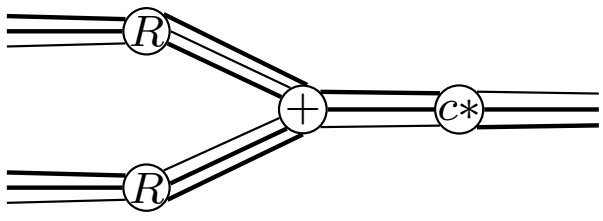

(t, p, ε)-RPC :

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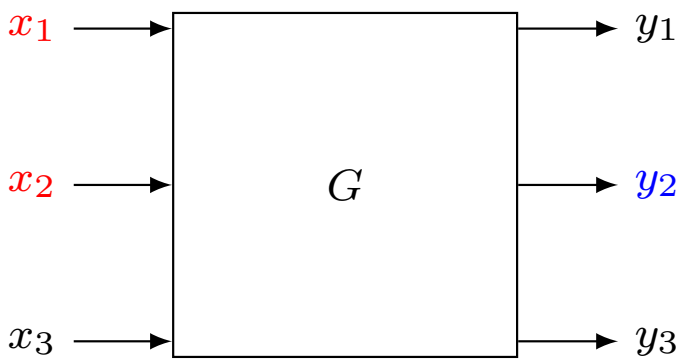
- Error probability ε.

General Random Probing Composability

- **Probe Distribution Table :** $\varepsilon_J(I) =$ Upper bound on the probability that we requires the set I of input shares to simulate the internal leakages and the output shares in a set J.
- **Example :**



I \ J	$\emptyset$	$\{y_1\}$	$\{y_2\}$	$\{y_3\}$	$\{y_1, y_2\}$	$\{y_1, y_3\}$	$\{y_2, y_3\}$	$\{y_1, y_2, y_3\}$
$\emptyset$								
$\{x_1\}$								
$\{x_2\}$								
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$\{x_1, x_3\}$								
$\{x_2, x_3\}$								
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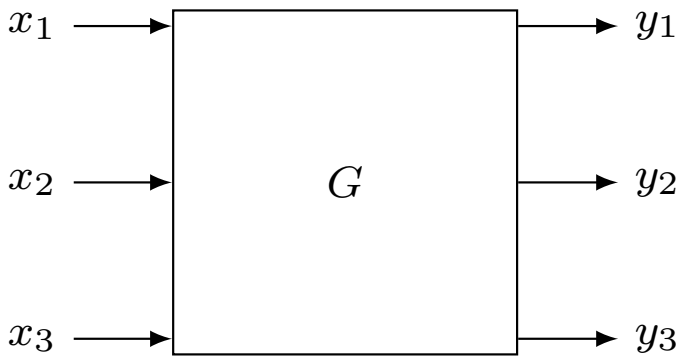
Cardinal Random Probing Composability

- $\varepsilon_{t_{out}}(t_{in})$ = Upper Bound on the probability that we requires t_{in} input shares to simulate the internal leakages and t_{out} output shares.



t_{in} \ t_{out}	0	1	2	3
0				
1				
2				
3				

Tradeoffs between the security notions

- Tighter security Bounds : General RPC $>$ Cardinal RPC $>$ Threshold RPC.
- Verification Complexity : General RPC $<$ Cardinal RPC $<$ Threshold RPC.

Uniform Cardinal Random Probing Composability

- Best of both world : Tight security from General RPC and complexity from Cardinal RPC.
- One Condition :

$$\varepsilon^{g}_{(J_1,\ldots,J_m)}(I_1,\ldots,I_\ell) = \varepsilon^{g}_{(J'_1,\ldots,J'_m)}(I'_1,\ldots,I'_\ell)$$

when

$$(|I_1|,\ldots,|I_\ell|) = (|I'_1|,\ldots,|I'_\ell|)$$
$$(|J_1|,\ldots,|J_m|) = (|J'_1|,\ldots,|J'_m|)$$

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0				
1				
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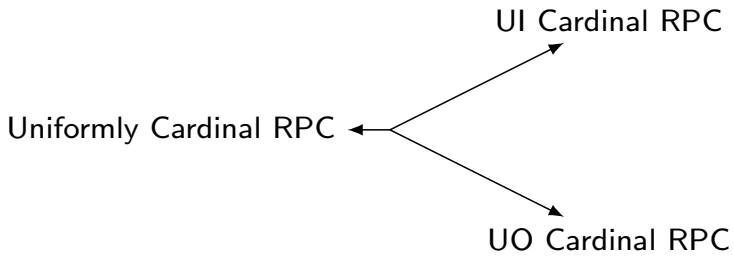
3				

Our contribution

- Revisit the notion of uniformly cardinal RPC.
- Method to transform any gadget satisfying cardinal/general RPC into a uniformly cardinal RPC.
- Design a non-linear multiplication gadget and prove its (uniformly) cardinal RPC security.
- Design a compiler, apply it to the AES circuit, and compare it to state-of-the-art methods in terms of the achievable security-performance trade-off RPC security

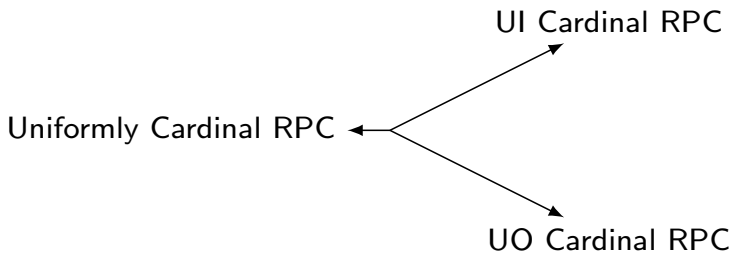
UI/UO cardinal RPC



- UI cardinal RPC, uniformity on the input shares **only**.

$$\varepsilon^g_{(J_1,\ldots,J_m)}(I_1,\ldots,I_\ell) = \varepsilon^g_{(J_1,\ldots,J_m)}(I'_1,\ldots,I'_\ell)$$

UI/UO cardinal RPC



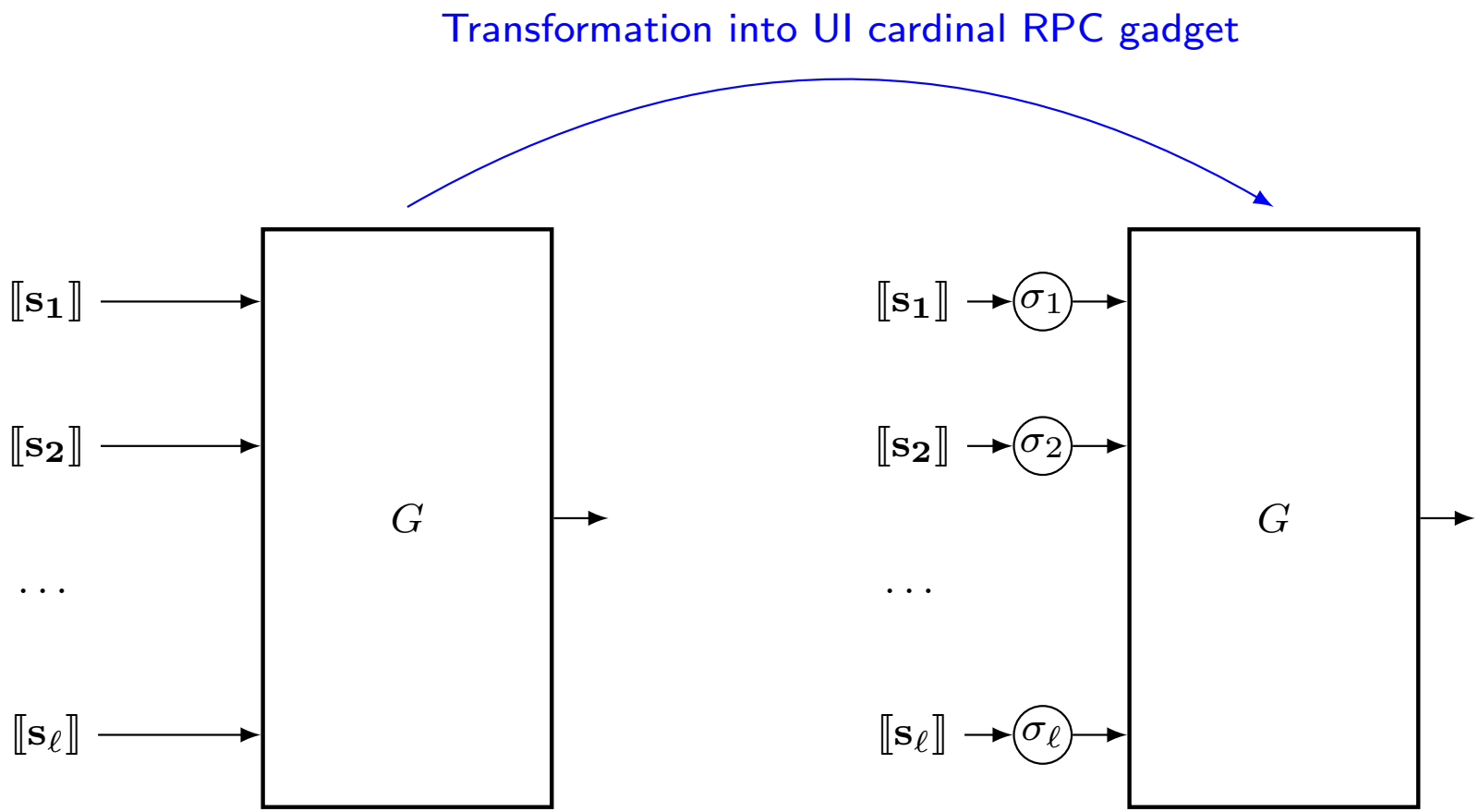
- UI cardinal RPC, uniformity on the input shares **only**.

$$\varepsilon^{g}_{(J_1,\ldots,J_m)}(I_1,\ldots,I_\ell) = \varepsilon^{g}_{(J_1,\ldots,J_m)}(I'_1,\ldots,I'_\ell)$$

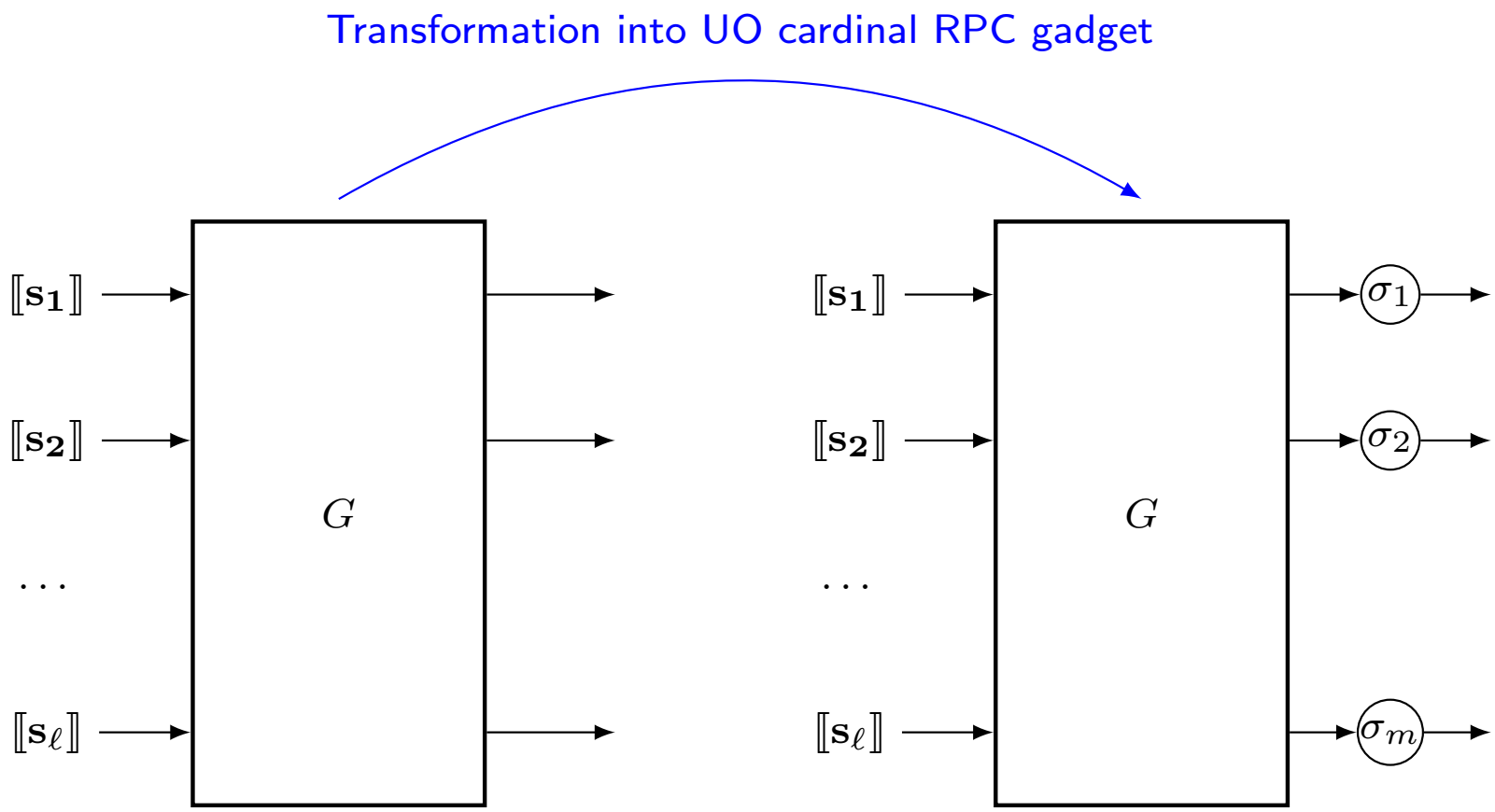
- UO cardinal RPC, uniformity on the output shares **only**.

$$\varepsilon^{g}_{(J_1,\ldots,J_m)}(I_1,\ldots,I_\ell) = \varepsilon^{g}_{(J'_1,\ldots,J'_m)}(I_1,\ldots,I_\ell)$$

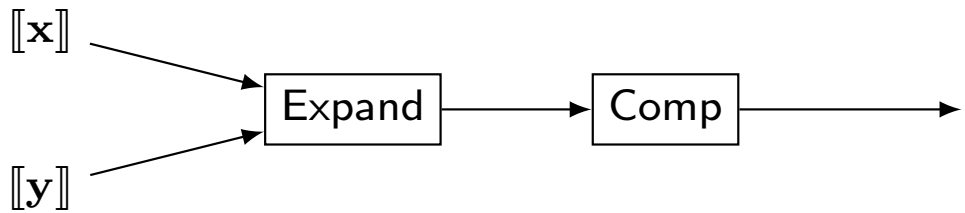
Put a gadget UI cardinal RPC



Put a gadget UO cardinal RPC

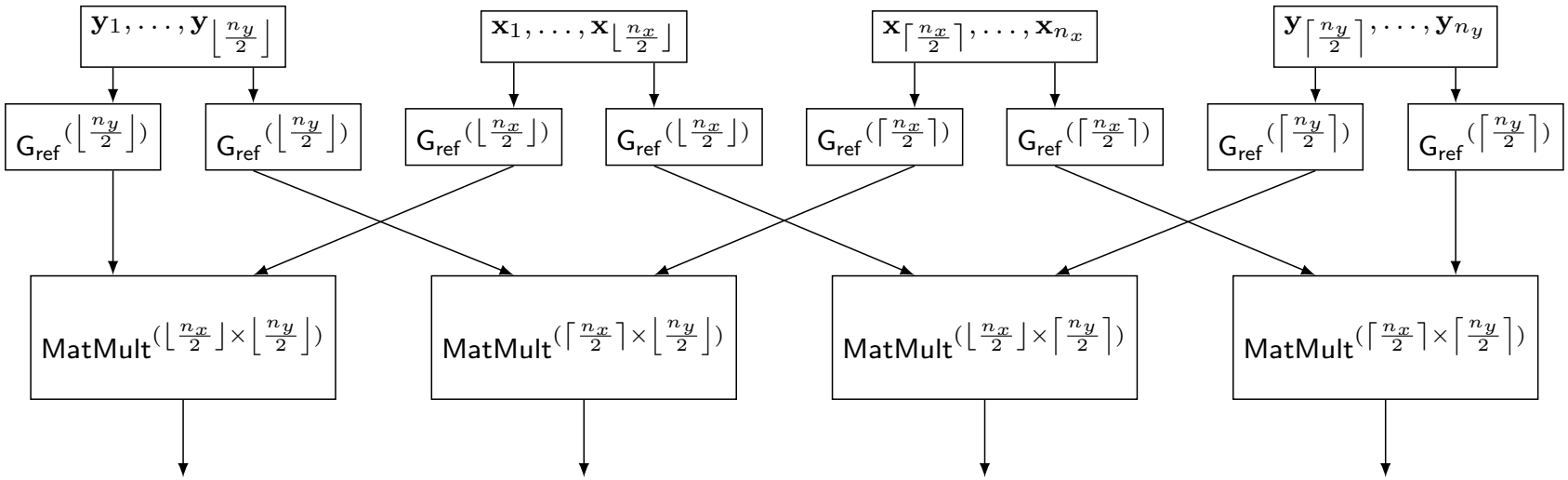


Multiplication gadget



- Expand : From two n-sharing, compute the cross product $x_i \cdot y_j$ to obtain a n^2 sharing.
- Comp : Transform a n^2-sharing into a n-sharing.

Expand : MatMultSym



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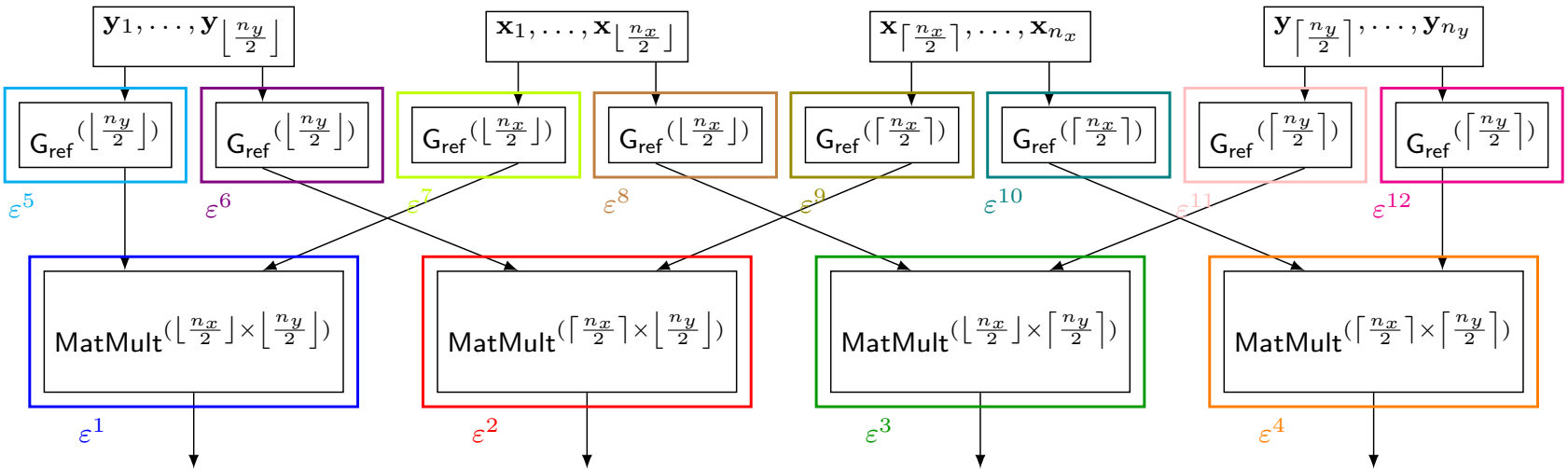
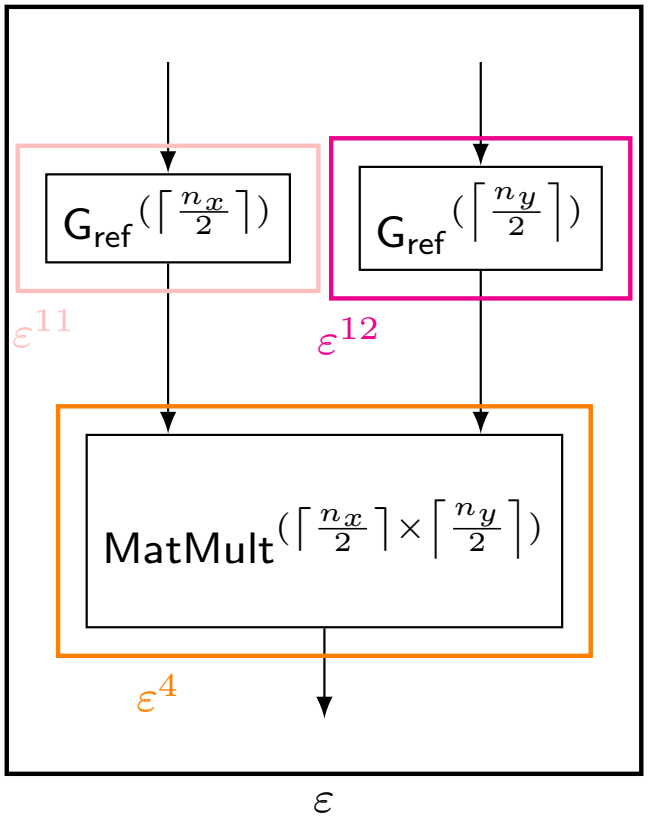
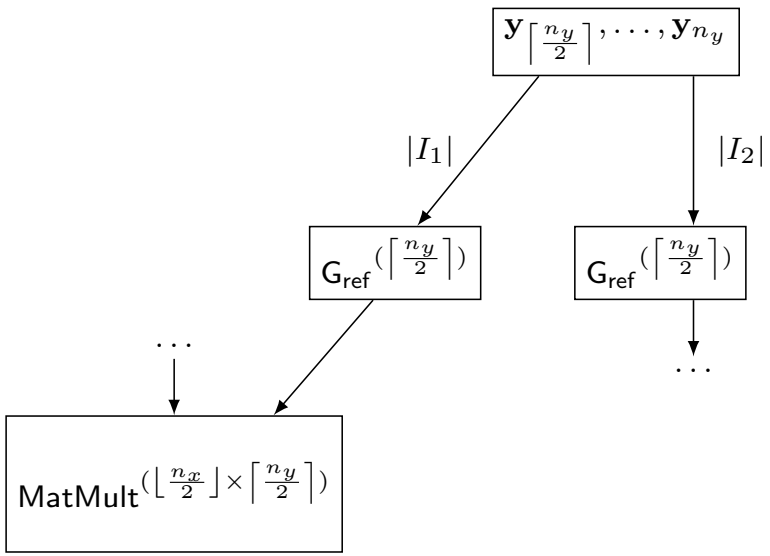


Expand : MatMultSym- Branch



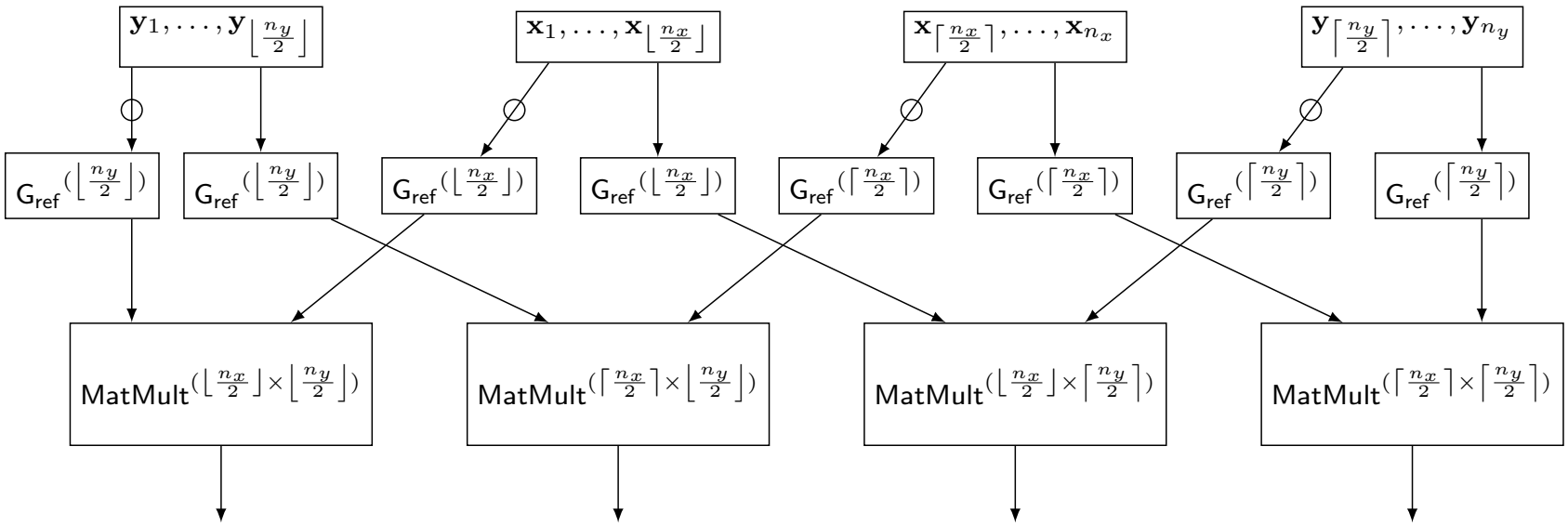
$$\varepsilon_j(i_1, i_2) = \sum_{\ell_1=0}^{n} \sum_{\ell_2=0}^{n} {\varepsilon^{11}}_{\ell_1}(i_1) \cdot {\varepsilon^{12}}_{\ell_2}(i_2) \cdot {\varepsilon^4}_j(\ell_1, \ell_2)$$

Expand : MatMultSym- Drawbacks

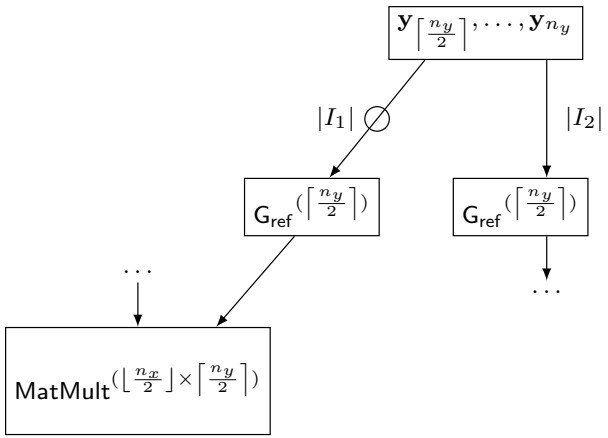


- $|I_1|$ required shares for the simulation of the first branch.
- $|I_2|$ required shares for the simulation of the second branch.
- **No informations** on $|I_1 \cup I_2|$.
- Must consider the worst case: $|I_1 \cup I_2| = \min(n, |I_1| + |I_2|)$.
- **Looser Bounds**

Expand : UnifMatMult



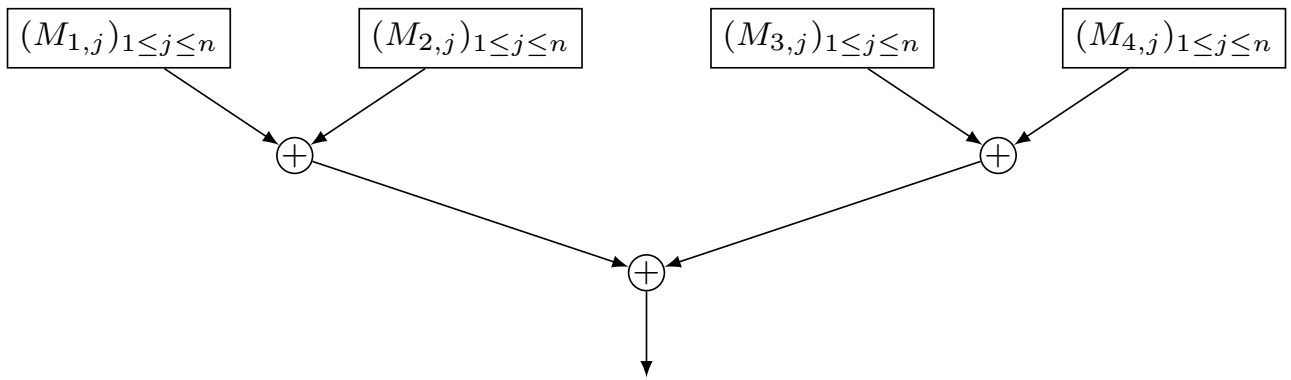
Expand : UnifMatMult



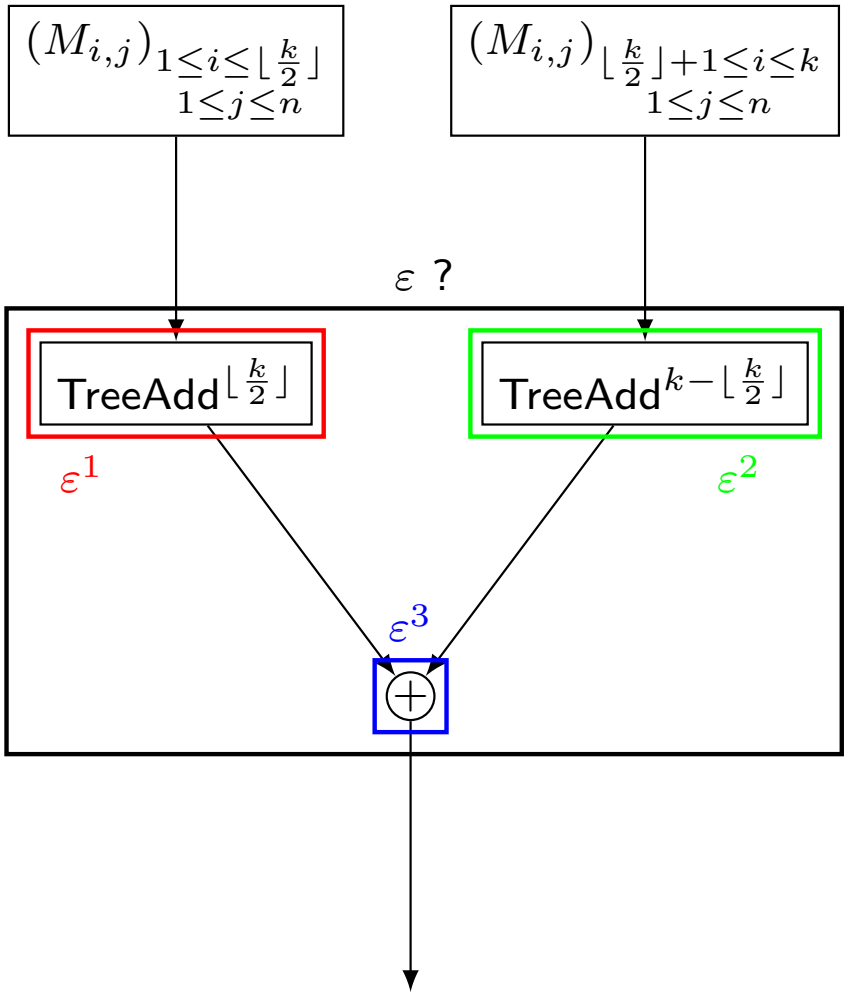
- **Informations** on $|I_1 \cup I_2|$.
- $\mathbb{P}(|I_1 \cup I_2| = \ell, |I_1| = \ell_1, |I_2| = \ell_2) = \frac{\binom{\ell_1}{\ell_1+\ell_2-\ell} \cdot \binom{n-\ell_1}{\ell-\ell_1}}{\binom{n}{\ell_2}}$
- **Tighter Bounds**

Comp : TreeAdd

- Transform a n^2 sharing into a n-sharing
- Tree of n-share gadget addition.
- Example with $n = 4$, and a $(M_{i,j})_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}}$ the n^2-sharing :

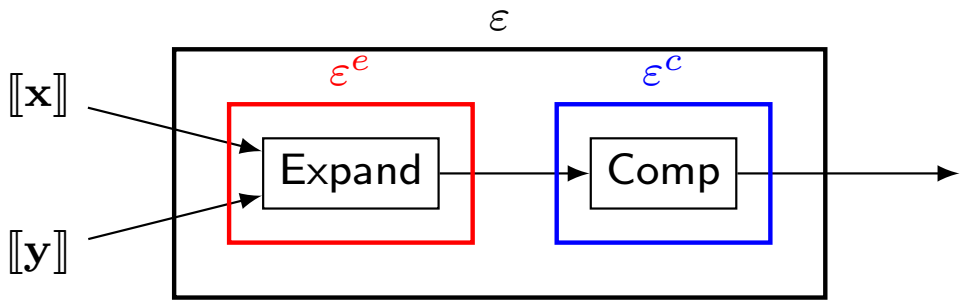


Comp : TreeAdd



$$\varepsilon_j(i) = \sum_{i_1+i_2=i} \sum_{\ell_1=0}^{n} \sum_{\ell_2=0}^{n} {\varepsilon^1}_{\ell_1}(i_1) \cdot {\varepsilon^2}_{\ell_2}(i_2) \cdot {\varepsilon^3}_{j}(\ell_1, \ell_2)$$

Multiplication gadget

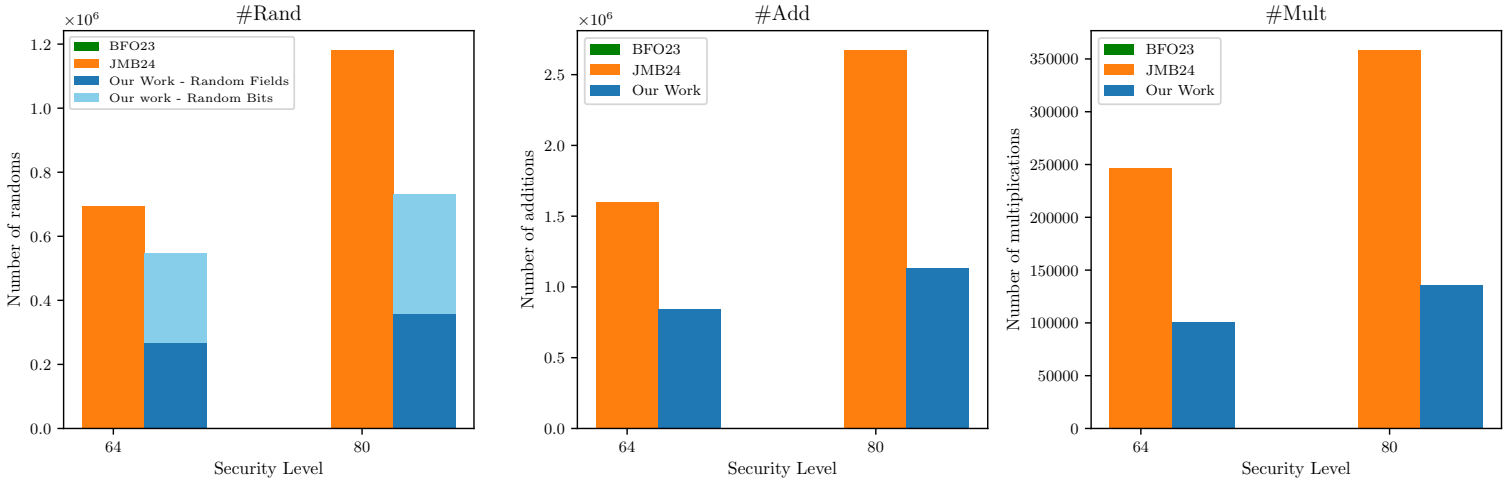


$$\varepsilon_j(i_{\mathbf{x}}, i_{\mathbf{y}}) = \sum_{\ell=0}^{n^2} {\varepsilon^e}_\ell(i_{\mathbf{x}}, i_{\mathbf{y}}) \cdot {\varepsilon^c}_j(\ell)$$

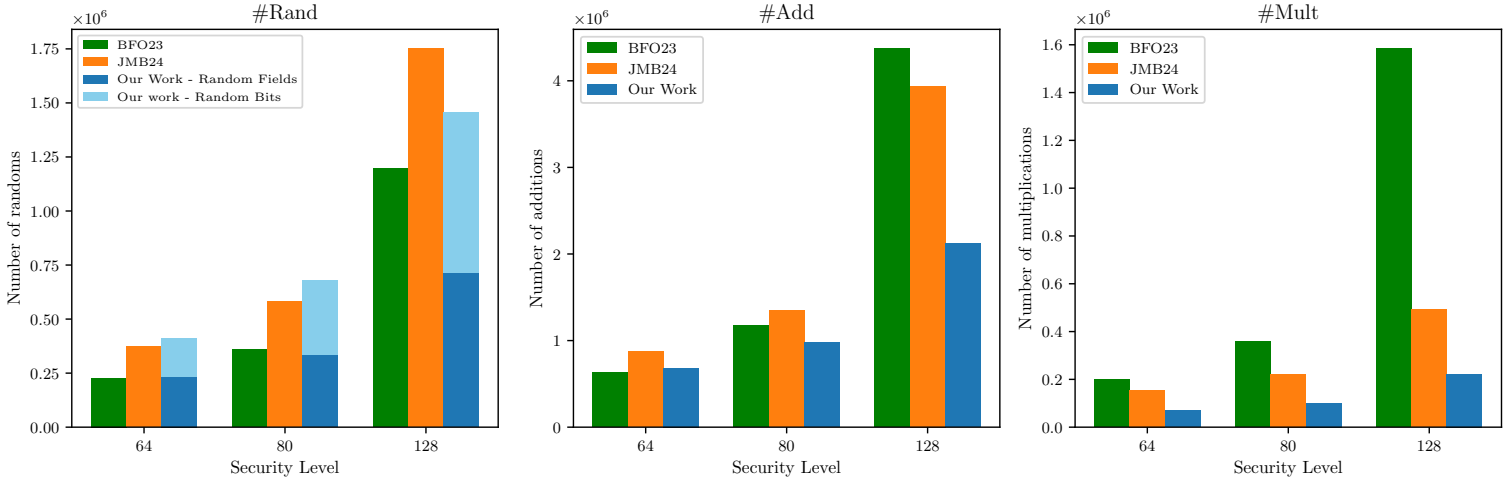
Compiler

- Linear Gadgets : $\mathsf{Add}^\gamma, \mathsf{cMult}^\gamma, \mathsf{cAdd}^\gamma \rightarrow$ Known cardinal RPC enveloppes.
- Squaring Gadget : In case $\mathbb{K} = \mathbb{F}_{2^k}$, $\mathsf{Sq}^\gamma \rightarrow$ Known cardinal RPC security enveloppes.
- Non Linear Gadget : Multiplication gadget used with UnifMatMult $\rightarrow$ Now known cardinal RPC security enveloppes.

AES Results



(a) $p = 2^{-16}$



(b) $p = 2^{-20}$

Thank You for your attention !